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Dedicated to mathematics in general and to the following aims in particular: (1) a study of the common problems of secondary and collegiate mathematics teaching, (2) a true valuation of the disciplines of mathematics, (3) the publication of high class expository papers on mathematics, (4) the development of greater public interest in mathematics by the publication of authoritative papers treating its cultural, humanistic and historical phases.

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A PRELIMINARY REPORT ON HOMOGENEOUS GROUPING IN FRESHMAN MATHEMATICS

By PROFESSOR JOSEPH SEIDLIN

The department of mathematics at Alfred University is attempting the experiment of sectioning its Ceramic College freshmen into homogeneous groups. At the end of mid-semester examinations and ratings, about the middle of last November, we resectioned 87 freshmen as follows:

Group X (30 students), consisting of the "A" and solid "B" students;

Group Y (32 students), consisting of the low "B", "C", and high "D" students;

Group Z (25 students), consisting of the low "D", "E", and "F" students.

We are herewith submitting our tentative conclusions in the hope that teachers of college mathematics interested in ways and means of improving teaching may, wherever practicable, join us in the experiment. At the end of two or three years by pooling our joint experiences we should be able to arrive at some definite conclusions and recommendations.

The Report of Professor Alfred E. Whitford

It fell to my lot to take a class of 30 freshmen, who had been graded either "A" or "B plus" at mid-semester. I am quite pleased with the results, both measurable and otherwise, attained thus far by this "select" group. I discovered the ability of the class as a whole to enter into discussion of mathematical theory with commendable thoroughness. I have been experiencing the unique satisfaction of having practically every student in the class participate intelligently in the discussion. All developmental procedure is speeded up since little time needs to be spent in answering questions of slower students, whose belated thinking requires a repetition of the argument. Again, it has been refreshing to have so unusually vivacious response by so large a group to an unusual "turn" in a problem or in a demonstration. Finally, and perhaps of greatest educational significance, the com-

position of the group enabled me to offer suggestions and ask questions involving broader and deeper aspects of the subject, resulting, it seems to me, in arousing a *genuine enthuiasm* for mathematics for its own sake. I am greatly interested in continuing the experiment for further testing of results.

The Report of Professor Lester R. Polan

The "middle" group remained remarkably average to the end of the first semester. With the beginning of the second semester, however, I have noticed a greater variation in quiz grades. In analytic geometry, in particular, several students have outgrown their former rank and almost all of the students have shown improvement. Personally, I have enjoyed teaching the group. I believe, too, that my teaching has been more effective. I feel that I have a clearer "teacher's conscience". Never before had I experienced that satisfied feeling of "reaching" the whole class. There have been very few embarrassing moments, for in nearly all cases the questions asked by the students relate to difficulties that are common to most of them. I am especially encouraged by rather frequent waves of enthusiasm not at all common in average students.

The Report of Professor Joseph Seidlin

I must admit that I experienced a rather uneasy feeling in facing a class of students who normally are not expected to survive. My feelings must have been akin to those of a nurse assigned to a case of a dying patient, whose days are numbered despite best intentions and care.

The first week was one of mutual adjustment. Evidently the students too were interested in their new environment. But hardly a week had passed when there came a deluge of questions. We had begun the topic of "Determinants"; we slid back to ninth grade algebra; we struck bottom at the foot of third grade arithmetic; and, in due time, returned to determinants. In such fashion we continued all of the first semester. We declared "open season" on all kinds of questions. There followed nine weeks of the most revealing "confessionals" in all of my teaching experience.

In the mid-year examinations my group averaged 60%; group Y averaged 68%; group X averaged 83%. Of the chartered 25

students of group Z, 23 survived. At the present writing,—just a week prior to our final examinations, I find that 3 of the group are potential "A" students, 4 are promising "B" students, 9 are "C" students, 6 range from high to low "D", one "dropped" the course.

Summary

We make no pronouncements. Frankly, we had not expected such "favorable" results. We realize, however, that all sorts of accidents and coincidences may have been at large to account for a good share of the "saving of souls". Needless to add that at Alfred we shall continue the experiment. We hope that, encouraged by even our very scant data, departments of mathematics in other colleges will attempt sectioning their freshmen into homogeneous groups. We recommend, however, that the sectioning be done not so much on the students' high school records as on the record of the first two months in college mathematics.

We made no attempt to cover the same ground in the same way in the three sections. Each of us employed his own schemes of presenting and developing subject matter. In group Z, without any formal planning on my part, I came to employ what educationists would call the Unit Plan. Since few questions pertaining to arithmetic, algebra, or geometry, relevant to the topic under consideration, are likely to be overlooked by the Z-group students, each topic needs must be treated as a complete whole. General practice of homogeneous grouping may necessitate textbooks especially prepared for the different groups. However, I want to warn professional textbook writers not to attempt to perpetrate a textbook for group Z until they have experienced the shocks and revelations attendant upon the actual teaching of group Z.

In conclusion may I emphasize the nature of this report. We are in the midst of an experiment. We are not submitting data for statistical analyses, nor are we issuing any proclamations on detailed treatment. We are merely sharing the *general nature* of our experiences for whatever they may be worth to fellow teachers of mathematics. True, we are a bit enthusiastic over the first gross results of our ex-

periment. We are encouraged to continue the experiment in the hope that ultimately it may result in:

(1) sustaining and promoting the interest and enthusiasm of the excellent and good students.

(2) developing the "mathematical abilities" in, and obtain maximal results from, the average students

(3) discovering the weak spots and eliminating prejudices, occasioned by former unfortunate contacts with earlier mathematical "instruction", of students who ordinarily fall by the wayside, victims of the motto: "They Shall Not Pass."

MATHEMATICAL PRODIGIES

By LOUIS McCREERY
University of California at Los Angeles

Mathematical prodigies are those gifted children who, when very young, startle those about them with their ability to calculate, do mathematical tricks or reason and learn, along mathematical, classical and mechanical lines, far beyond the average child of the same age. Theirs are the names which blaze in the mathematical sky with great initial brilliance. Some are meteors which flash across the heavens, attracting much attention and acclaim, but soon turning to ashes and vanishing. Others blaze brilliantly and remain steady as stars contributing much, finally fading with many years of research a service to their credit. In this latter never forgotten group we find such men as Euler, Hamilton, Leibniz, Gauss, Clairaut and Maclaurin. In the former group we find lightning calculators whose feats astounded the world, but who contributed little or nothing to the furtherance of the science of mathematics.

Let us first consider the meteors, those interesting men at whom we marvel for their uncanny ability to add large numbers, to extract cubes and squares, without the aid of pencil or paper. Have these minds contributed anything to mathematics or are they merely freaks whose abilities have made a place for them on the vaudeville stage?

The first mention we have of a lightning calculator whose powers appeared at an early age is Zerah Colburn, who lived from 1804 to 1840. His father was a Vermont farmer, and there is no record of genius in the family.

At the age of seven, at which time Zerah had had only six weeks schooling, he immediately gave the correct answer to the multiplication 13×97. His powers seem to have been developing for at least six months before their discovery, for he may have learned numbers from his elder brothers and sisters. When Zerah was eight years old, his father took him to London, where among other feats he found mentally the 16th power of 8, a 15 digit number. Also in London he learned to read and write, and here he began the study of Algebra.[1] In his many times interrupted schooling, he showed no great ability along any other line. In fact "Colburn's calculating powers, . . . seemed to have absorbed all his mental energy; he was unable to learn much of anything, and incapable of exercise of even ordinary intelligence or of any practical application."[2] Colburn served some time as a computer in the service of the secretary of the Board of Longitude. He then returned to the United States, and made a small living as a minister and a teacher in the foreign languages.

There are others who showed this remarkable power, but all followed the same lines until Johann Martin Zacharias Dase used his powers for some permanent good. Dase, a German who lived from 1824 to 1861, made many friends among Mathematicians and they led him to devote his powers to the service of science. He calculated the value of Π to 200 decimal places by the formula

$$\frac{\Pi}{4} = \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{5}$$

in two months; computed 7 place logarithms for numbers from 1 to 1,050,000; and constructed factoring tables for numbers for the 7th and 8th millions, and parts of the 9th and 10th millions. He had planned to finish these tables, but death cut short his labors.[3] When he asked the Academy of Sciences at Hamburg for support in this work, they consented, provided Gauss would agree. Gauss answered saying: "With small numbers, everybody that possesses any readiness in reckoning, sees the answer to such a question (the divisibility

[1]Mitchell, Frank D. *Mathematical Prodigies*, American Journal of Psychology, Vol. 18, pg. 66.
[2]Scripture, E. W. *Arithmetical Prodigies*, American Journal of Psychology, Vol. 4, pg. 16.
[3]Mitchell, op. cit., pg. 77.

of a number) at once directly, for greater numbers with more or less trouble; this trouble grows in an increasing relation as the numbers grow, till even a practiced reckoner requires hours, yes days, for a single number; for still greater numbers, the solution by special calculation is entirely impracticable . . . . . You possess many of the requisite qualities (for establishing a table of factors) in a special degree, a remarkable agility and quickness in handling Arithmetical operations . . . . . and an invulnerable persistence and perseverence."[4] This was a real recommendation, for Gauss was a man of exceptional calculating ability himself.

Another class of lightning calculators consists of those who used their power in science. Among these is George Parker Bidder, a son of a stone-mason of Devonshire, England. The indications of hereditary influence are stronger in the Bidder family than in any of the other formerly considered. Bidder's eldest brother had a remarkable memory, another was an excellent mathematician and an insurance actuary, and his sisters possessed powers of mental arithmetic above the average. Another fact concerning Bidder is the use to which he put his powers. After leaving Edinburgh, he devoted himself to civil engineering, and was connected with several engineering projects of the first magnitude. "As a member of the Institution of Civil Engineers he (Bidder) took a prominent part in the controversies then before the profession. Constant use kept alive his calculating powers, and in various railway and other contests before great Parliamentary Committees his great command of statistics and keen powers of analysis made him a formidable witness."[5] For example: "Two days before his death the query was suggested that taking the velocity of light at 190,00 miles per second, and the wave length of the red rays at 36,918 to an inch, how many of its waves must strike the eye in one second. His friend, producing a pencil, was about to calculate the result, when Mr. Bidder said, "You need not work it, the number will be 444, 433,651,200,000."[6] One can imagine how such powers would be of use to an engineer in a Parliamentary Committee room. In fact many plans for railways and like projects were rejected when he found flaws in the plans and calculations that would take an ordinary man many days to discover. As may be seen from the above Bidder was interested in general principles, practical applications and striking

[4]Scripture, op. cit., from Gauss' letter in the Preface to Dase's Factoren-Tafeln, 7th millicn, Hamburg, 1862.
[5]Mitchell, op. cit., pg. 81.
[6]Scripture, op. cit., pg. 25.

properties, rather than in intricate analysis for its own sake, or calculations with numbers chosen merely for their length. So he became an engineer rather than a pure mathematician like Hamilton.

Bidder's son, G. P. Bidder, Jr., surpassed his father in length mental multiplication, but lagged behind in accuracy and rapidity. The younger Bidder was 7th wrangler at Cambridge, distinguished himself in mathematics, later became a thriving barrister and Queen's Counsel. He used his powers in cryptography. His place in mathematics is small, for he directed his abilities into other lines.

A case somewhat like that of the elder Bidder is that of Henry Truman Safford, who lived from 1836 to 1901. He was the son of a Vermont farmer, but both parents had been school teachers and were well educated. The father had a strong interest in mathematics, and the mother was of "exquisite nervous temperament." Safford was a precocious youngster in almost every line, and his amazing ability to calculate was discovered when he was three years old. His most amazing feats were accomplished when he was still a boy. At seven he had equalled Zerah Colburn's mental computations, and had started to study algebra and geometry. He soon passed to higher mathematics; and needing some logarithms he found them himself by formulae! At the age of ten he published an almanac computed entirely by himself. At thirteen he devised a method for finding the time of an eclipse which reduced the labor 25 per cent.[7] As his early inclinations seemed to lead, he became an astronomer, but his success in later life was not as exceptional as that of his youth.

The mental calculator who contributed the most to the sciences is Carl Friedrich Gauss, a German who lived from 1777 to 1855. His father was a day laborer, but there seems to be some mathematical genius in his family. It is said of him that he could reckon before he could talk. He seemed to have a "peculiar sense for the quick apprehension of the most complicated relations of numbers and an unsurpassed memory for figures. At the age of ten he entered higher analysis, and distinguished himself in the sciences at college.

The prodigies heretofore studied seemed to have been limited in their fields. That is, they became famous in only one field of scientific endeavor. But Gauss enriched nearly every kind of mathematics; particularly the theory of numbers, which calls for the ability to perceive relations between numbers, an ability Gauss had to perhaps a greater degree than the world has produced before or since. Mathe-

[7]Scripture, op. cit., pg. 36.

matics is indebted to him, outside of his work in the theory of numbers, for the idea of theory of least squares, work on the theory of surfaces, complex numbers, congruences, and hyperbolic geometry. An example of his work is the proposition that a circle can be divided into seventeen equal parts by Euclidean methods.[8] He also enriched the mathematics of astronomy, geodesy, and electricity. Small wonder then that Kronecker said of him "Almost everything which the mathematics of the century has brought forth in the way of original scientific ideas is connected with the name of Gauss."[9]

Then there is Ramanujan of India, who is a pure genius. The fact that he arose from a country long dormant in mathematical activity further proves the statement that for genius there is no explanation. Ramanujan was born in 1887 in a humble Indian home. He started to school at the age of five, and at the age of seven startled his teachers with questions about "zeros, imaginary quantities, and distances of stars." He did not play with the rest of the children, but spent his time in verifying formulae in the theory of numbers. Young Ramanujan dropped out of college in his junior year because of failures in English, but he did not give up mathematical research. Out of school he could work in the way in which he was most productive—strictly by intuition. He recognized general formulae in number, which he noted. Sometimes he proved these formulae, more often not. Ramanujan died in 1920, and English mathematicians are slowly proving these formulae.[10]

While we have no record of his being an exceptional calculator, he possessed a wonderful memory and knew the idiosyncracies of number to a remarkable degree. For instance, when Ramanujan was in England, Professor G. H. Hardy remarked that the number 1729 was an exceedingly dull one. "On the contrary", replied the Indian mathematician, "it is an exceedingly interesting one, for it is the first number that is expressible as the sum of two cubes in two different ways."[11]

We find among these lightning calculators men whose genius was limited merely to number manipulation, and men whose minds could embrace science and mathematics in all its higher branches. There is some indication that if genius or superior ability had been found in the family before, the one so endowed would become more

[8]Smith, D. E., *A History of Mathamtics*, Vol. I, Ginn and Co. 1923, pg. 504.
[9]Ibid., pg. 504.
[10]Srikantia, B. M. *Srinvasa Ramanujan*, American Mathematical Monthly, Vol. 35, pgs. 241-5.
[11]Ibid.

than a mere calculator, but as in most studies of human beings the data is incomplete and insufficient, and there are always exceptions.

Now there is another kind of genius who astounds those about him when very young. His ability to see relations of form, to analyze and generalize, stand out.

Perhaps the outstanding example of this type is Blaise Pascal, a Frenchman who lived from 1623 to 1662. He was the son of an able mathematician who could and did start him in the right direction. Blaise showed phenomenal ability at an early age, and "although his father wished him first to have a thorough grounding in the ancient languages, and therefore took from him all books, he succeeded in beginning geometry all by himself."[12] He obtained from his father a definition of geometry, then drew figures on the wall of the nursery. He devised means of constructing a perfect circle and an equilateral triangle without aid. When his father "discovered him he had proved the first 31 theorems of Euclid. He played with conics as other children play with toys, but with the divine enjoyment of discovering eternal truths."[13] At sixteen he wrote on conics and at nineteen he invented the first computing machine. Before seventeen he wanted to know a reason for everything and had an unusually keen eye for falsity. As Bidder discerned mistakes in number, Pascal perceived error in form.

One of his most important discoveries is Pascal's theorem: the three points determined by producing the opposite sides of a hexagon inscribed in a conic are collinear,—a theorem from which he deduced four hundred corollaries. He is noted for his work in Pascal's triangle, on the theory of probability, the theory of the cycloid and the problem of its general quadrature, and in projective geometry. His work, in the main, followed his early inclunations.

There are mathematical prodigies whose abilities, while not taking so definite a form as in the case of Pascal, were very apparent. The most outstanding of these is Sir William Rowan Hamilton (1805-1865). Young William, who sprang from an intellectual family, had a command of English and arithmetic at the age of three years, was able to read and translate Latin, Greek, and Hebrew at five, and at eight could talk Italian and French freely, and give vent to his feelings in extemporized Latin. At the age of twelve, he met Zerah Colburn in a series of mental calculating contests, which he generally

[12]Smith, op. cit., pg. 382.
[13]Ibid.

lost, but showed himself with honor. This gave him a taste for computation which he kept throughout his life. When he was thirteen he studied Clairaut's Algebra in the French, and made an epitome which he ambitiously entitled "A Compendious Treatise on Algebra by William Hamilton".[14] At college Hamilton received the most unusual distinction of winning highest honors in the classics and mathematics.

His life of great contributions to mathematics and allied sciences began at the age of sixteen when he detected a flaw in LaPlace's reasoning in his "Mechanique Celeste". At twenty-two he was appointed professor of astronomy at the University of Dublin. He soon led the field of mathematicians, which is to be expected of a man capable of such mental calculations, and possessing such an analytic sense. His great discovery was the theory of Quaternions, but during his life he enriched nearly every branch of mathematics.

A case similar to that of Hamilton though not so remarkable is that of William Leibniz, a German who lived from 1646 to 1716, and who came from a good family. Young Leibniz was familiar with several languages before he was twelve, and "found his most absorbing childhood interests in reading, at first history and poetry, later philosophy and logic. He sought at a very early age to make his knowledge correct and ready, and complete by classifying and systematizing it by using signs and characters in place of words, by generalizing and by bringing every inquiry under a principle and a method."[15] Small wonder he was drawn to mathematics through philosophy, his first love. With this description of his powers as a child, we do not wonder that he introduced more lasting symbols than and other mathematician, that his idea of calculus was the more logical and useful, and that he early made a name for himself in the mother of all sciences, philosophy.

Hamilton and Leibniz belonged to the class of pure mathematicians. Their philosophy regarding the sciences is shown by a remark made by one of them in discussing a certain method, "It is the peculiar beauty of this method, gentlemen, and one which endears it to the really scientific mind, that under no circumstances can it be of the smallest possible utility."[16]

[14]MacFarlane, Alexander, Ten British Mathematicians of the 19th Century, New York, John Wiley and Sons, Inc., 1916, pg. 35.
[15]Cox, G. M. Genetic Studies of Genius, Stanford University Press, 1925.
[16]Smith, op. cit., pg. 467.

There is still another class of prodigies, composed of men who would take issue with the above remark. Their genius was expressed early in the field of art, music, or inventions; and their mathematical accomplishments were generally (one must always qualify his generalization when he is writing concerning human beings) along practical lines. Such men as Leonardo da Vinci (1452-1519), Galileo (1564-1642), and Isaac Newton (1642-1727) belonged to this group.

Sir Isaac Newton belongs in this classification for his youthful genius, which by the way, was not as great as that of some of the others, was expressed by work with his hands, directed, of course, by his brain, but the result was a physical object not an idea. He invented toys, constructed working models of windmills which were excellent, made kites for his friends after investigating the best forms and proportions, and constructed small chairs and tables for a feminine friend. Before he was seventeen, after watching the movements of the sun, he constructed three unusually accurate sun dials.[17] At first glance it would seem that Newton's contributions to mathematics were pure like those of Hamilton. However we must remember that Newton's contributions to physics and astronomy at least equalled his contributions to mathematics.

Da Vinci's early genius first showed itself in art. He soon branched out, however, and became famous in art, science, and study of the human body. Smith states that: "In geometry he distinguished between curves of single and double curvature, gave much attention to the subject of stellar polygons, was interested in the constructions with a single opening of the compasses, and gave various correct or approximate constructions of regular polygons." He seems to have taken little interest in the question of the cubic which was occupying the minds of the mathematicians of his day. He was interested in mathematics because of the constructions and its applications, not because the problem was of "no possible utility."

Galileo was the son of an Italian cloth merchant, who was a student of mathematics and music. Galileo showed remarkable intellectual aptitude, and mechanical invention from earliest childhood. His favorite pastime was the construction of original and ingenious toy machines; but his application to literary studies was equally great. In musical skill and inventions he early vied with the best professors in Italy, and his art ability attracted much attention.[18]

[17]Encyclopedia Britannica, 11th edition.
[18]Encyclopedia Britannica, 11th edition.

At the age of seventeen he made his all important discovery of the fact that a pendulum swings with equal oscillations. He had been entirely ignorant of mathematics up to that time, but he inadvertently overheard a lecture by Ricci, and immediately became engaged in the study of mathematics. His important works are in the applications of mathematics and the invention of the proportional compasses.

Youthful geniuses thus seem to be divided into three different classes: those who were exceptional calculators, those who were exceptional geometers and algebraists, and those whose genius first, showed itself in scientific lines. The powers of the first group lay in seeing relations of number; the powers of the second and third groups in seeing relations of forms. But any attempt to classify human nature must be very general and flexible, for there is always the exceptional case who will arise and upset the grouping. Hamilton, for example, belonged in each of these groups.

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*THE FOUNDING OF NON-EUCLIDEAN GEOMETRY

By P. H. DAUS
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The ever recurring newspaper publicity given to the founder and the subject matter of the theory of relativity has made the public acquainted at least with the term non-Euclidean geometry. It is the purpose of this short talk to tell something of one of the most interesting chapters in the history of geometry, a chapter concerned with the attempts, failures and successes to either vindicate or improve what Euclid wrote twenty-two centuries ago, a chapter which even today is being amended with the application of non-Euclidean geometry to our modern physical world.

Euclid of Alexandria, whose influence on mathematics began about 300 B. C., is perhaps the best known of all writers connected with the School of Alexandria. He bears the unique distinction of being the only man to whom there ever came or ever can come again the glory of having successfully incorporated in his own writings all the essential parts of the accumulated mathematical knowledge of his time. He was the most successful textbook writer that the world has ever known; his Geometry has gone through more than a thousand editions and, although considerably changed in form, it is still in use

*Prepared for presentation over the radio, March 25, 1933.

in our schools today. No doubt a considerable part of the book was original with Euclid, but more important is the fact that he set himself the task of organizing and coordinating in a simple logical sequence the sum total of all geometric knowledge then extant. How well he succeeded is attested by the words of a later day famous geometer, W. K. Clifford: "This book has been for nearly twenty-two centuries the encouragement and guide of that scientific thought which is one thing with the progress of man from a worse to a better state."

Logical sequence was predominant throughout the work. Euclid attempted to state explicitly as an axiom or postulate every fact which he could not deduce from others. He took nothing for granted, except that which he stated would be taken so. He recognized that certain facts, like "If equals be added to equals the results will be equal", would be accepted as self-evident truths, and these he called axioms. Other facts which were essential to the development of geometry and which it seems impossible to suppose that Euclid imagined to be self-evident, he called postulates. The most famous of these is known as the *parallel postulate*. Although not stated in Euclid's words, several forms of the postulate are these. If two straight lines in a plane are both perpendicular to a third line, they will never meet no matter how far produced, that is, the lines are parallel. Or, one and only one straight line can be drawn through a point parallel to a given straight line. Or the parallel postulate. may be replaced by one that appeals more directly to physical experiment or our intuitions: the sum of the angles of a triangle equals 180 degrees; or again, straight lines which are everywhere equidistant are parallel, and still other forms. But all forms of the parallel postulate or its substitute contain a direct or an implied assumption, and it is greatly to Euclid's credit that he explicitly recognized this fact.

For two thousand or more years after Euclid, many real and psuedo-Mathematicians tried to prove this postulate, that is, they tried to derive this assumption from the others. Many false proofs were published, but we now know that all such attempts were doomed to failure. The first lasting contribution was made by an Italian Jesuit, Giovanni Girolamo Saccheri, whose works were first published after his death in 1733, under the title *Euclidis ab omni naevo vindicati*, (*Euclid freed of every blemish*). If Saccheri had had a little more imagination and had been less bound down by tradition and a firmly implanted belief that Euclid's hypothesis was the only true one, he

would have anticipated by a century the discovery of non-Euclidean geometry.

For Saccheri tried to discover what would happen if Euclid's parallel postulate were rejected. He set up in its stead two alternatives, known as the hypotheses of the acute angle and obtuse angle, respectively. In terms of the angles of the triangle, they assume that the sum of the angles of a triangle is less than or greater than 180 degrees, respectively. He found little difficulty in demolishing the hypothesis of the obtuse angle. It led to a contradiction of one of the propositions of Euclid, whose proof did not require the parallel postulate The contradiction obtained here, however, is not valid, because of several implied assumptions in Euclid's work. One of these is that a straight line is of infinite length, an assumption which the Theory of Relativity has forcibly called to the attention of the scientific and even the general public. With the other hypothesis, Saccheri was not so successful. He gave a long so-called proof that this hypothesis must also be rejected, but not satisfied with this attempt he offered another "proof" in which he lost himself in the quicksands of the infinitesimal and in philosophical meanderings. It was not a logical but a psychological difficulty, which caused Saccheri to reject the very conclusions to which his own labors clearly and inevitably pointed. Euclid vindicated, when the blame was never Euclid's!

But interest in the problem did not lag. About 1800 the genius of Gauss was being attracted to the question, and although he published nothing bearing directly on the question, it is clear from his correspondence that he was deeply interested in the subject While at first he was inclined to the orthodox belief, encouraged by Kant, that Euclidean geometry was an example of a necessary truth, he was probably the first to obtain a clear idea of the possibility of a geometry other than that of Euclid, and we owe the very name non-Euclidean geometry to him. At that time the theory of parallels was in disrepute, and anyone working in that field was deemed queer. Gauss lacked the convictions and courage necessary to publish his discoveries, and he was finally forestalled by receiving from his friend and former fellow-student, Wolfgang Bolyai, a copy of the now famous Appendix by his son, John Bolyai.

The honor for the discovery of non-Euclidean geometry belongs simultaneously to a Russian Professor of Mathematics, Nikolai I. Lobachevsky, and to a young Hungarian student, John Bolyai. They both arrived at their conclusions independently of Gauss and of each

other. Lobachevsky was interested in the theory of parallels as early as 1815. In 1823 he prepared a treatise on geometry for use in the University, but it was received so unfavorably that it was never printed, until the manuscript was rediscovered and printed in 1909. In 1826 he presented to the physical and mathematical section of the University of Kazan, a paper in which he developed a system of geometry which is equivalent to that of Saccheri under the hypothesis of the acute angle but he recognized that this geometry was just as consistent as that of Euclid. Thereafter he wrote several extensive memoirs expounding the new geometry.

Working independently, John Bolyai, while a student at the Royal College of Engineers at Vienna, communicated to his father in 1823, the main idea of the non-Euclidean geometry he had discovered, but the results were not published until 1832. Again the geometry he had discovered was also equivalent to that of Saccheri under the hypothesis of the acute angle.

The ideas inaugurated by Lobachevsky and Bolyai did not attain any wide recognition for many years. It was not until after Riemann's death and the publication in 1866 of his now famous dissertation of 1854, that non-Euclidean geometry began to be seriously studied, even by mathematicians; and in recent times, until the discovery by Einstein of the Theory of Relativity, it has had no serious recognition by other scientific scholars. It is remarkable that after the publication of the works of Lobachevsky and Bolyai, it never occurred to any of those working in this field to examine more carefully the hypothesis of the obtuse angle. As already indicated this involves the assumption that a straight line is unbounded but yet of finite length. This concept we owe to Bernhardt Riemann, who in his Dissertation of 1854 laid the analytical foundation for non-Euclidean geometry and our present day Theory of Relativity.

Imagine a sphere, say the earth, with the equator and a number of meridians drawn. All of these meridians are perpendicular to the equator; they all meet and have a finite length. This picture gives a crude idea of the situation in such a non-Euclidean geometry, a geometry, which, with its refinements and extensions to space-time, becomes the geometry of our modern universe in the Theory of Relativity.

At last mathematicians and other scientists have come to realize that the parallel postulate of Euclid is not capable of proof; that an equally consistent geometry can be developed if we suppose that it is

not true and replace it by another hypothesis. The term "self-evident truth" is no longer in good standing. We now recognize that all the axioms and postulates of Euclid are after all only hypothesis which our limited experience has led us to accept as true, but which could be replaced by contrary statements in the development of another logical geometry. And further, that these new geometries may equally well find applications to our physical world, just as the geometry of Riemann has found its place in the Theory of Relativity.

THE CARTESIAN TANGENT CLUB

Dear Reader:

I have found Descartes' method of tangents very interesting and instructive not only to myself but to my students. In sending in my exposition of this method, I hope I may entice you to join me in making Descartes' method known to students in the classroom and especially at club meetings.

For the purpose of extending the knowledge of Descartes' method of tangents to a wide circle of teachers and students, I should like to form *The Cartesian Tangent Club*. Write me a postcard and I shall acknowledge your membership with thanks and without any dues.

WILSON L. MISER,
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May 9, 1933.

DESCARTES' METHOD OF TANGENTS

By WILSON L. MISER

In his La Geometrie (1637) Rene Descartes gave methods for examining the nature and properties of curved lines. In the latter part of the second book*, he says: "I shall have given a sufficient introduction to the study of curves when I have given a general method of drawing a straight line making right angles with a curve at an arbitrarily chosen point upon it. And I dare say that this not only

*Page 95, The Geometry of Rene Descartes, Translated from the French and Latin by D. E. Smith and Marcia L. Latham. The Open Court Publishing Co., Chicago, 1925.

the most useful and most general problem in geometry that I know, but even that I have ever desired to know."

It is the purpose of this paper to explain Descartes' method in the manner and notation of the analytic geometry of today. Before proceeding to a description of the method, consider the following example:

Example 1. Find the equation of the tangent to the parabola

(1) $$y^2 = x$$

at the point P.

Suppose that the problem is solved and that the normal PN is drawn in Fig. 1. Let $ON = v$, the intercept of the normal with the $x-$axis. We proceed now to find v in terms of x.

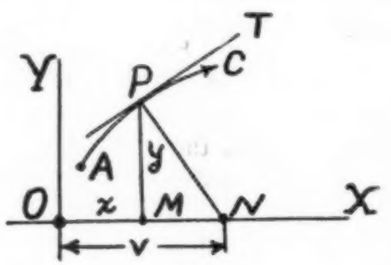

Fig. 1.

Let $PN = s$. Then $(v-x)^2 + y^2 = s^2$ which may be written in the standard form of the equation of a circle

(2) $$(x-v)^2 + y^2 = s^2,$$

having the center N(v,O) and the radius r.

Regarding equations (1) and (2) as simultaneous, the abscissas of the points of intersection of the parabola and the circle are given by the quadratic equation

(3) $$x^2 + (1-2v)x + v^2 - s^2 = 0,$$

since by equation (1) y^2 can be replaced by x in equation (2).

Evidently there is but one normal to the parabola at the point P and so the equation (3) must have equal roots. Let each root $x = e$, then the equation (3) is in the form

(4) $$x^2-2ex+e^2=0.$$

By equating the coefficients of x in equations (3) and (4),

$$1-2v=-2e,$$

whence $$v=e+\tfrac{1}{2}$$

or $$v=x+\tfrac{1}{2},$$

for $e=x$. Hence v, the intercept of the normal, is expressed in x.

The slope of the normal PN is $\dfrac{y-0}{x-v}=-2y$. The slope of the tangent is $\tfrac{1}{2}y$, the negative reciprocal of $-2y$. When this slope is evaluated for a certain point (x',y') on the parabola, the equation of the tangent at that point of contact is

$$y-y'=(\tfrac{1}{2}y')(x-x')$$

which reduces to

$$2y'y=x+x',$$

the usual form of the equation.

A description of the method. Let APC be a curve for which the algebraic equation

(i) $$f(x,y)=0$$

is given in rectangular coordinates. To find the equation of the tangent PT at the point P, suppose that the normal PN is drawn as in Fig. 1 and that the normal cuts the x-axis at the point N. Letting $ON=v$, $PN=s$, the problem is to express v in terms of x.

Since PMN is a right triangle,

(ii) $$(x-v)^2+y^2=s^2.$$

Regarding equations (i) and (ii) as simultaneous, y can be eliminated so as to obtain an algebraic equation in x, say

(iii) $$F(x)=0.$$

Since there is but one normal PN to the curve at the point P, at least

two roots of equation (iii) are equal. By equating the coefficients of equation (iii) to the corresponding coefficients of the equation

(iv) $$(x-e)^2(p+qx+rx^2+\ .\ .\ .\ )=0,$$

it is possible for many curves to express v in terms of x so that the slope of the normal PN can be found. Finally the equation of the tangent PT can be written from its slope and the coordinates of the point P of contact.

Example 2. Find the equation of the tangent to the ellipse

(1) $$x^2+4y^2-4x=0$$

at a given point P.

Sketching the ellipse and supposing that the tangent and normal are drawn in a figure very much like Fig. 1, the equation

(2) $$(x-v)^2+y^2=s^2$$

becomes, by substituting from equation (1) for y^2,

(3) $$(x-v)^2+x-\frac{1}{4}x^2-s^2=0.$$

Dividing equation (3) by $\frac{3}{4}$ when the terms are collected and equating the coefficient of x to the coefficient of x in the equation

(4) $$x^2-2ex+e^2=0,$$

we have $$\frac{4}{3}(1-2v)=-2e$$

whence $$v=\frac{3}{4}x+\frac{1}{2},$$

for e may be replaced by x.

The slope of the normal PN is $(y-0)/(x-v)$ which equals $4y/(x-2$. The slope of the tangent is $-(x-2)/4y$. Hence the equation of the tangent at the point (x', y') of contact is

$$y-y'=((-x'+2)/4y')(x-x')$$

which simplifies by using equation (1) to

$$(x'-2)x+4y'y-2x'=0.$$

Example 3. Find the equation of the tangent to the equilateral hyperbola

$$xy=1 \tag{1}$$

at a given point.

Drawing the hyperbola in the first quadrant and supposing that the normal PN is drawn to the hyperbola at P as in Fig. 2,

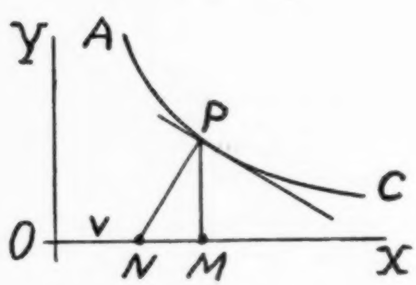

Fig. 2.

the equation

$$(x-v)^2+y^2=s^2 \tag{2}$$

becomes, by substituting from equation (1) for y^2,

$$(x-v)^2+1/x^2=s^2.$$

This equation simplifies to the biquadratic in x

$$x^4-2vx^3+(v^2-s^2)x^2+1=0. \tag{3}$$

Equating the coefficients of (3) to the corresponding coefficients of the biquadratic

$$(x^2-2ex+e^2)(p+qx+rx^2)=0 \tag{4}$$

which has two roots $x=e=e$, we have

$$r=1,$$

$$-2er+q=-2v,$$

$$e^2r-2eq+p=v^2-s^2,$$

$$e^2p=1.$$

Eliminating p and q and using the relations $x=e$ and

$$v^2-s^2=2vx-x^2-1/x^2,$$

we find $$v=x-1/x^3.$$

The slope of the normal is $(y-0)/(x-v)$ which equals x^2. The slope of the tangent is $-1/x^2$. Hence the equation of the tangent at the point (x',y') of contact is

$$y-y'=(-1/x'^2)(x-x')$$

which reduces to

$$y'x+x'y=2.$$

In order that the reader may have some simple examples very much like the examples worked out, the author suggests

(a) The equilateral hyperbola, $x^2-y^2=1$,

(b) The parabola, $y=4x-x^2$,

(c) The cubic, $y=x^3$, and

(d) The semi-cubical parabola, $y^2=x^3$.

ON THE SIMPLIFICATION OF FORMULAS IN THE MATHEMATICS OF FINANCE

By CLIFFORD BELL
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During the past several years a marked tendency towards simplification of formulas has developed among writers on the mathematics of finance. The most outstanding step in this direction has been the adoption, by some of the prominent text book writers, of the interest conversion period as the unit of time rather than the long used year period. Thus if P units of principal are invested at the actual rate of i per period for n interest conversion periods, the amount S, at the end of that time, is given by the single formula, $S=P(1+i)^n$. The value of this simplification can hardly be overestimated, since the

compound interest law is the basis of most of the formulas in the mathematics of finance.

It is believed that much can be done both in the way of presenting and of simplifying the question of annuities. It is quite the usual thing for a student of finance to become very enthusiastic over the simple applications of the compound interest law. He even masters the all important principle of equivalence, which states that different sums of money due at different dates can be added only if brought to a common time. Following the usual plan of presenting annuities, the student's enthusiasm is very likely to be dampened when a mass of symbols, such as $RS_{\overline{n}|}$, $Ra_{\overline{n}|}$, $RS^{(p)}_{\overline{n}|}$, $Ra^{(p)}_{\overline{n}|}$, i/j_p, is thrust upon him. He is also perplexed upon being told that R, in the symbols $RS^{(p)}_{\overline{n}|}$ and $Ra^{(p)}_{\overline{n}|}$, stands for the sum of the p separate payments made in one interest conversion period. Even after being assured by his instructor that the value of R, so defined, is the result of purely algebraic work, the student is still likely to feel that the principle of equivalence has been violated.

The author has found that the student is less confused by the following method of presentation. Let the given annuity, whether an ordinary annuity or an annuity due, be replaced by an equivalent annuity whose payments are made at the end of each interest conversion period. Let R be the size of each of these payments and let n be the term expressed in interest conversion periods. The accumulation and present value are then given respectively by the symbols $RS_{\overline{n}|}$ and $Ra_{\overline{n}|}$. No doubt as to the validity of the principle of equivalence arises by this method of presentation, as the student must use this very principle in order to determine R in most of the cases that occur.

The student will have no difficulty in setting up the formulas for R that are given in some recent texts, such as C. H. Forsyth: Introduction to the Mathematical Theory of Finance, Wiley, 1928. Thus if K is the ratio of the length of the payment interval to the length of the interest conversion period and if X is the annuity payment, then $R = X/S_{\overline{k}|}$ for ordinary annuities and $R = X/a_{\overline{k}|}$ for annuities due. Hence if k is an integer the values of $1/a_{\overline{k}|}$ and $1/s_{\overline{k}|}$ may easily be obtained obtained from the $1/a_{\overline{n}|}$ table, given in all text books. However no current text gives values of $1/a_{\overline{k}|}$ when n is fractional. But since it is easily shown that $1/a_{\overline{1/p}|} = pi/j_p + i$, the computations can be carried out even for the fractional values by making use of the table always given for i/j_p, p being the reciprocal of the above-mentioned k. Furth-

ermore, as $1/a_{\overline{n}|} = 1/s_{\overline{n}|} + i$, for both integral and fractional values of n, $1/s_{\overline{1/p}|}$ is given by p times the table value of i/j_p.

It should be pointed out that any trouble the student may have with annuities, as presented under the above system, arises from the difficulty involved in computation, rather than from a misconception of the theory. Until a table is made, including the most used fractional values of n, this computational difficulty will naturally persist. It is believed that the greater ease with which the student grasps the subject warrants the inclusion of entries for fractional values of n in the $1/a_{\overline{n}|}$ table. Such entries could easily be computed from the i/j_p table as indicated above and as the i/j_p table would no longer be needed, the total space used for tables would remain the same. It is hoped that such entries will appear in any new tables that may be made in the near future.

LINEAR EQUATIONS IN A CERTAIN CHEMICAL ANALYSIS

By S. T. SANDERS, Jr.
St. Joseph, Mo.

An interesting setting for a system of linear equations is found in a phase of the new process of gasoline manufacture.

Crude oil, at a temperature of 860° F., is placed in contact with hydrogen gas at a pressure of 3000# per sq. in. Chemical changes involving the carbon of the oil and the hydrogen produce gasoline, some of the carbon, however, uniting with the hydrogen to produce gaseous hydrocarbons. The efficiency of this "cracking" process being determined in part by the nature and relative volume of these hydrocarbons, it is customary to analyze the gas escaping from the still.

The analyst strips the gas successively of carbon dioxide, unsaturated hydrocarbons, carbon monoxide, and hydrogen, preparatory to measuring the hydrocarbons. Then the latter, upon union with oxygen, are burned out and the resultant loss in volume is recorded. The volume of carbon dioxide evolved during the combustion is now determined by noting the loss of gas on contact with an alkaline solution.

Letting C_c and C_a represent respectively the combustion contrac-

tion in volume and the alkaline contraction, we may write for each volume present of C_mH_{2m+2} and $C_n H_{2n+2}$:

$$C_m H_{2m+2} + \tfrac{1}{2}(3m+1)O_2 \rightarrow mCO_2 + (m+1)H_2O \tag{1}$$

$$C_n H_{2n+2} + \tfrac{1}{2}(3n+1)O_2 \rightarrow nCO_2 + (n+1)H_2O \tag{2}$$

The reduction in (1) on combustion is seen to be

$$1+\tfrac{1}{2}(3m+1)-m=\tfrac{1}{2}(m+3)$$

That is,

$$\begin{aligned} C_c &= \tfrac{1}{2}(m+3)C_m H_{2m+2} + \tfrac{1}{2}(n+3)C_n H_{2n+2}, \\ C_a &= m\, C_m H_{2m+2} + n\, C_n H_{2n+2}, \end{aligned} \tag{3}$$

where the variables on the right now represent volumes of the particular gases.

Solving the system, (3):

$$\begin{aligned} C_m H_{2m+2} &= \frac{2nC_c-(n+3)C_a}{3(n-m)} \\ C_n H_{2n+2} &= \frac{(m+3)C_a-2mC_c}{3(n-m)} \end{aligned} \tag{4}$$

Now in practice it usually happens that $n=m+1$, in which case (4) reduces to:

$$\begin{aligned} C_m H_{2m+2} &= \tfrac{1}{3}[2(m+1)C_c - (m+4)C_a]; \\ C_{m+1} H_{2m+4} &= \tfrac{1}{3}[(m+3)C_a - 2mC_c]. \end{aligned} \tag{5}$$

Indeed, $m=1$ is the situation that the analyst most often meets; that is, a combination of methane and ethane.

$$\begin{aligned} C H_4 &= \tfrac{1}{3}(4C_a - 5C_c) \\ C_2 H_6 &= \tfrac{1}{3}(4C_a - 2C_a) \end{aligned} \tag{6}$$

From (4) follows the total volume of hydrocarbons:

$$C_m H_{2m+2} + C_n H_{2n+2} = \tfrac{1}{3}(2C_c - C_a) \tag{7}$$

It is to be noted, in closing, that this method seldom yields true results, since it is rare that only two hydrocarbons are present in the gas to be analyzed. And, of course, a unique determination of the volumes of three or more hydrocarbons is impossible, since they are expressed in terms of only two parameters. However, from (7) it follows that the latter definitely fix the total volume of the hydrocarbons, regardless of their number.

THE HYPERGEOMETRIC OF GAUSS

(Continuing Article in January Number)

By ABE HACKMAN

11. In studying the convergence of the hypergeometric series, we shall make use of the following principles and tests.

A. The *necessary* test for convergence requires that the nth term of the series approach 0 as a limit as n increases indefinitely.

B. The necessary test is also *sufficient* if the series is composed of terms which are alternately positive and negative, but it is not sufficient if all the terms in the series are of the same sign. (For example, the harmonic series, $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots$ is divergent as it stands, but the series, $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots$ is convergent.)

C. A series composed of complex terms (of the form $u+v\sqrt{-1}$) is convergent when the series composed of the moduli (of the form $\sqrt{u^2+v^2}$) of the terms is convergent.

D. A series is convergent if the modulus (absolute value, i. e. modulus of -4 is 4, modulus of $\sqrt{-4}$ is 2, modulus of $3+\sqrt{-16}$ is $\sqrt{25}=5$) of the ratio of the $(n+1)$st term to the nth term becomes less than a quantity which is less by a finite amount than unity, as n increases indefinitely. If, as n increases indefinitely, the modulus of the ratio remains greater than unity or approaches unity from the upper side, the series is divergent. If, as n increases indefinitely, the modulus of the ratio approaches unity from the lower side, this test fails to tell whether the series is convergent or divergent. This test is known as the first test ratio test.

E. A series of positive terms is convergent if the following condition is satisfied.

$$n\left(1-\frac{W_{n+1}}{W_n}\right)$$ must be greater than unity as n increases indefinitely. The ratio here is by its very nature positive, since the test applies to a series of positive terms. This is the second test ratio test.

F. A series of positive terms is convergent if the following condition is satisfied

$$n \log n-(n+1) \log (n+1) \frac{W_{n+1}}{W_n}$$

must be greater than O, as n increases indefinitely. This is the third test ratio test.

12. Let us apply the first test ratio test to the series. As in 3, we shall use as the test ratio of the series, the ratio between the (m+2)nd term to the (m+1)st for the sake of convenience. Since the test applies when the number of the term increases indefinitely, this change is legitimate. We shall employ this same convention thruout the discussion. The symbol | | means modulus thruout.

$$\left|\frac{W_{n+1}}{W_n}\right| = \left|\frac{(\alpha+m)(\beta+m)}{(1+m)(y+m)} x\right| \quad (13)$$ or dividing thru by m^2

$$= \left|\frac{\dfrac{\alpha\beta}{n^2}+\dfrac{\alpha+\beta}{m}+1}{\dfrac{y}{n^2}+\dfrac{1+y}{m}+1}\right| |x|$$

The first factor has the limit unity as m increases indefinitely or

$$\left|\frac{W_{x+1}}{W_x}\right|_{n\to\infty} = |x| < 1 \quad (14)$$ for convergence.

From (14) we conclude that when the modulus of x, the variable of the hypergeometric is less than unity, the series is convergent; when it it is greater than unity, the series is divergent.

13. We have now to consider what happens when the modulus of x is equal to unity. In this case the first test ratio test tells us nothing as to the convergence of the series. It now becomes a problem of determining what relation must exist between the parameters of the series, so that the series may be convergent when the variable is equal to +1 or −1. Let us divide the problem into these two parts, and consider first the case where $x = +1$.

14. Let us now apply the second test ratio test to the series for the case where $x = +1$. In this case we have a series, in which the x terms, if not from the beginning, then after some definite number has been reached, are all positive. The second and third test ratio tests, therefore apply.

$$n\left(1-\frac{W_{n+1}}{W_n}\right) = m\left(1-\frac{(\alpha+m)(\beta+m)}{(1+m)(y+m}\right)$$

$$= m\,\frac{m^2+(y+1)m+y-m^2-(\alpha+\beta)m-\alpha\beta}{m^2+(y+1)m+y}$$

Dividing numerator and denominator by m^2, we get

$$n\left(1-\frac{W_{n+1}}{W_n}\right) = \frac{y+1-\alpha-\beta+\dfrac{y-\alpha\beta}{m}}{1+\dfrac{y+1}{m}+y/m^2}$$

the limiting value of which, as m increases indefinitely, becomes $y+1-\alpha-\beta$. For convergence this quantity must be greater than unity or $\alpha+\beta-y<0$. If $\alpha+\beta-y>0$, the series is divergent (15)

15. If $\alpha+\beta-y=0$ then the limit of the quantity in the second test ratio test becomes unity, and we must have recourse to the third test ratio test. In this test,

$$n\log n-(n+1)\log(n+1\frac{W_{n+1}}{W_n}=m\log m-(m+1)\log(m+1)\frac{(\alpha+m)(\beta+m)}{(1+m)(y+m)}$$

must be greater than 0, or positive.

This reduces to

$$\frac{(m^2+ym)\log m-(m^2+\alpha m+\beta m+\alpha\beta)\log(m+1)}{y+m}$$

But $\qquad \log(m+1)=\log m+\log(1+\frac{1}{m})$

$$=\log m+\left\{\frac{1}{m}-\frac{1}{2m^2}+\frac{1}{3m^3}-\frac{1}{4m^4}+\ldots\right\}$$

So that our expression becomes

$$\frac{[(\alpha+\beta-y)m-\alpha\beta]\log m-[m^2+(\alpha+\beta)m+\alpha\beta]\left\{\frac{1}{m}-\frac{1}{2m^2}+\frac{1}{3m^1}+\ldots\right\}}{y+m}$$

But $\alpha+\beta-y=0$. The expression becomes negative for indefinitely large values of m, whatever the signs or values of the parameters, which must be finite. Therefore when $x=+1$ and $\alpha+\beta-y=0$ the series is divergent.

16. To recapitulate: When the modulus of x is less than unity, the series is convergent. When the modulus of x is greater than unity, the series is divergent. When $x=+1$ and $\alpha+\beta-y$ is less than 0, the series is convergent. When $x=+1$ and $\alpha+\beta-y$ is equal to, or greater than 0 the series is divergent. The case when $x=-1$ may be treated in a later paper.

ON A FAMILY OF CURVES

By W. VANN PARKER

In Vol. 7, No. 5 of this News Letter, William Sell discusses the graph of the function defined by

$$y=f(x)=\begin{cases} \dfrac{U^{x+1}}{x+1} & (x\neq -1), \\ \log U & (x=-1), \end{cases}$$

for a fixed positive value of U, and calls attention to its resemblance to an equilateral hyperbola. If we consider U as a parameter varying through positive values, the family of curves thus obtained present several interesting features.

Let us leave aside for the moment the isolated point and consider the family of curves defined by

$$F(x,y)\equiv xy+y-U^{x+1}=0$$

as U assumes all positive values. We get at once

$$\frac{dy}{dx}=\frac{U^{x+1}[(x+1)\log U-1]}{(x+1)^2}.$$

For $U\neq 1$, this derivative is zero when

$$x=-1+\frac{1}{\log U},$$

and for this value of x

$$\frac{d^2y}{dx^2}=e\log^3 U.$$

We see, therefore, that for $0<U<1$, the curve has a maximum point to the left of $x=-1$; and for $U>1$, the curve has a minimum point to the right of $x=-1$. For $U=1$, the curve becomes the equilateral hyperbola

$$xy+y-1=0$$

As U varies through positive values, this hyperbola is a limiting position between those curves which have maximum points and those which have minimum points.

It is interesting to note also that the locus of the maximum and minimum points of this family of curves is also an equilateral hyperbola.

If $x=-1+\dfrac{1}{\log U}$, then $y=e\log U$, and if we eliminate U between these equations we get

$$xy+y-e=0$$

as the locus of the maximum and minimum points.

As U varies from zero, the isolated point moves up the line $x=-1$, and becomes the point $(-1,0)$ for $U=1$.

AN ELECTRICAL PROBLEM

By W. E. BYRNE

It is well known that the instantaneous value of the current I (amperes) in a circuit with resistance R (ohms) and inductance L (henries) in series is given by the equation

$$L\frac{dI}{dt}+RI=\varphi(t) \tag{1}$$

where $\varphi(t)$ is the applied electromotive force (volts) given as a function of the time t (seconds). Texts on alternating current theory take up

the discussion of this problem for the case of sinusoidal voltage waves, i. e.

$$(2) \qquad \varphi(t) = E \sin(\omega t + \alpha)$$

(E, ω, α constants.)

What is the result for arbitrary non-sinusoidal voltages, that is, for $\varphi(t)$ an arbitrary periodic integrable function? Furthermore, the general theory of linear differential equations shows that the solution of (1) consists of two parts: a *transient* part which tends toward 0 as time increases indefinitely (i. e. integral without second member) and a steady or permanent state part which is the limiting form of the current wave as t increases indefinitely. What is the shape of the steady state wave if $\varphi(t+T) \equiv \varphi(t)$, for instance if the graph of $\varphi(t)$ is like that indicated in Figure 1?

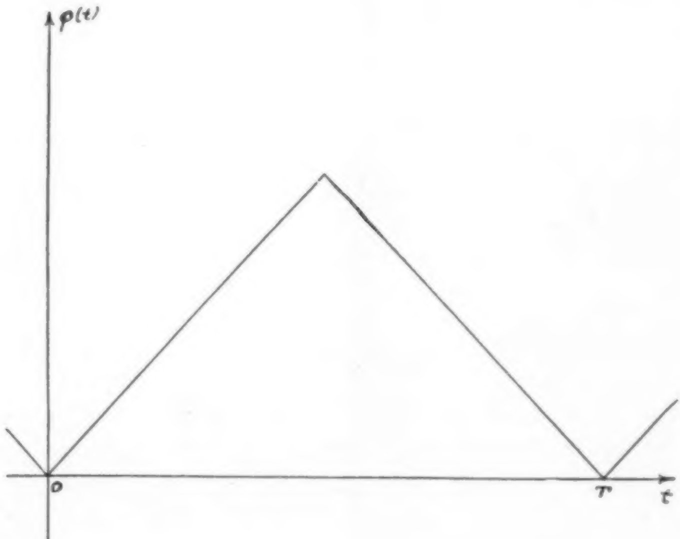

Figure 1

The solution of (1) is

$$(3) \quad I(t) = I_0 e^{\frac{-Rt}{L}} + \frac{1}{L} e^{\frac{-Rt}{L}} \int_0^t e^{\frac{Rt}{L}} \varphi(t)\,dt$$

where I_0 is the initial current. For purposes of notation let

$$I_n = I(nT) \text{ and } \alpha = e^{\frac{RT}{L}} \tag{4}$$

Then

$$I_n = I_0\alpha^{-n} + \frac{1}{L}\alpha^{-n}\int_0^{nT} e^{\frac{Rt}{L}}\varphi(t)dt \tag{5}$$

The transformation $t = \theta + nT$ (6)

applied to

$$\int_{nT}^{t} e^{\frac{Rt}{L}}\varphi(t)dt \tag{7}$$

yields

$$\int_0^{\theta} e^{\frac{R}{L}(\theta+nT)}\varphi(\theta+nT)d\theta = \alpha^n\int_0^{\theta} e^{\frac{R\theta}{L}}\varphi(\theta)d\theta \tag{8}$$

and so

$$I_n = I_0\alpha^{-n} + \tag{9}$$

$$\frac{1}{L}\alpha^{-n}\left[\int_0^{T} e^{\frac{Rt}{L}}\varphi(t)dt + \int_T^{2T} e^{\frac{Rt}{L}}\varphi(t)dt + \ldots + \int_{(n-1)7}^{nT} e^{\frac{Rt}{L}}\varphi(t)dt\right]$$

$$= I_0\alpha^{-n} + \frac{1}{L}\alpha^{-n}[1+\alpha+\alpha^2+\ .\ .\ .\ +\alpha^{n-1}]\int_0^{T} e^{\frac{Rt}{L}}\varphi(t)dt \tag{10}$$

$$= I_0^{-n} + \frac{1}{L}\frac{1-\alpha^{-n}}{\alpha-1}\int_0^{T} e^{\frac{Rt}{L}}\varphi(t)dt$$

It follows that

$$(11)\qquad \lim_{n\to+\infty} I_n = \frac{1}{L}\,\frac{1}{\alpha-1}\int_0^T e^{\frac{Rt}{L}}\varphi(t)\,dt$$

which is independent of I_0.

In order to analyze the permanent wave form make the transformation (6) on (1) and (3) to find:

$$(12)\qquad L\frac{dI}{d\theta}+RI=\varphi(nT+\theta)=\varphi(\theta)$$

$$(13)\qquad I(nT+\theta)=I_n e^{\frac{-R\theta}{L}}+\frac{1}{L}e^{\frac{-R\theta}{L}}\int_0^\theta e^{\frac{Rt}{L}}\varphi(t)\,dt$$

The same result could be obtained by the process of reasoning used in passing from (3) to (10). If now n is increased indefinitely,

$$(14)\qquad \lim_{n\to+\infty} I(nT+\theta)=\bar{\phi}(\theta)=(\lim_{n\to+\infty} I_n)e^{\frac{-R\theta}{L}}+\frac{1}{L}e^{\frac{-R\theta}{L}}\int_0^\theta e^{\frac{Rt}{L}}\varphi(t)\,dt$$

which gives the permanent wave form as θ varies from O to T.

Conclusion: The investigation of the permanent wave form requires the evaluation of (11) and sketching the result of (14). Mathematically the problem is that of determining the transform of a periodic function $\phi(t)$ by means of the formula

$$(15)\qquad \bar{\phi}(t)=\frac{e^{\frac{-Rt}{L}}}{L(\alpha-1)}\int_0^T e^{\frac{Rx}{L}}\varphi(x)\,dx+\frac{1}{L}e^{\frac{-Rt}{L}}\int_0^t e^{\frac{Rx}{L}}\varphi(x)\,dx$$

$$0 \ngtr t \ngtr T$$

BOOK REVIEW DEPARTMENT

Edited by P. K. SMITH

A First Course in Calculus. -By Edwin S. Crawley and Perry A. Caris. F. S. Crofts and Co., New York, 1933.

This text is written in clear and forceful English. The usual material to be found in a first course in calculus is covered. The author deviates some from most texts in calculus to give the same prominence to the differentiation formulas for the hyperbolic functions as he does to the trignonometric or the inverse trigonometric functions. The simple theorems on limits are proved in the first chapter. It is well to include these proofs in a course in calculus since a large percentage of students would never learn them elsewhere. The theory is amply illustrated with well selected problems. It is evident throughout the text that the authors are making all effort at clear exposition and attempting to select problems within the range of the student of average ability.

This text is well written and teachable. However, a few observations must be made by the teacher who desires to do his work in a critical manner. In the attempt to make clear the meaning of the differential of a function the authors make the geometrical approach giving the definition at the end. In the opinion of the writer the simpler approach is to give the definition $dy = F'(x)\Delta x$ and, then, will follow the significance of the derivative as a fraction and the geometrical meaning. The writers plunge too early into the use of the differential element in treating the definite integral as the limit of a sum. It is not shown that the differential element is primarily a mne monical tool rather than mechanism for a rigorous and understandable derivation of a formula.

On page 108 the writers in proving theorem VII on infinitesimals make the statement that

$$\lim \frac{\beta \pm \gamma}{\alpha^m} = \lim(a \pm b\alpha^P) = a,$$

where α, β, and γ are infinitesimals and a, b, P, and m are finite constants, follows from

$$\lim (\beta \pm \gamma) = \lim \alpha^m \cdot \lim (a \pm b)^P.$$

Evidently to derive the former equation the latter equation must be divided by $\lim \alpha^m$. This step is false since $\text{Lim } \alpha^m = 0$.

The first thirteen chapters are devoted entirely to differential calculus. The figures are excellent. The size is 6″x9″.—*P. K. Smith.*

Analytic Geometry. By Frederick S. Nowlan, Professor of Mathematics, University of British Columbia. McGraw Hill Book Co., New York, 1933. PP. 295. Price $2.25.

This text contains only plane analytic geometry. The subject is covered in twelve chapters with an additional chapter devoted to determinants and their use in solving systems of linear equations.

Soon after beginning to read this text the fact that a critical thinker is at work is easily sensed. Professor Nowlan has not written just another analytic geometry text. That this production is worthwhile is evidenced by uniqueness of presentation and strict adherence to rigor, at the same time, the author has not sacrificed simplicity to critical attack.

From the beginning orthogonal projection is used to derive many theorems in simpler manner. The statements of the fundamental concepts related to equations of loci and loci of equations are especially good. The conics are treated from the standpoint of the general definition. The distinguishing feature of the text is its use of the parametric equations of the straight line. In an elegant manner the parametric equations are used in deriving the equations of tangents, diameters, and polars of the conics. The harmonic—mean definition of the polar is given. In using the parametric equations the writer acquaints the student with certain methods in advanced geometry.

Twenty-six pages are devoted to the general equation of the second degree. The parametric equations of the straight are used in treating principal diameters, centers, and vertices of the conics. The subject of invariants connected with the general equation of the second degree is treated far more fully than is usually found in an elementary text on analytic geometric.

The author gives, in addition to the trigonometric, a non-trigonometric definition of the slope of a line. The object, he states, is to select a course without requiring a knowledge of trignonometry. If those parts of the course which use trignonometry were omitted, a very incomplete course would result. Perhaps, it would be better to devote the effort required in circumventing trigonometry, with a view to a short course, to the teaching of the elements of trigonometry. There is a strong question as to the wisdom of attempting analytic geometry prior to a course in trignonometry.

The observation must be made that the space given to transcendental curves is altogether too brief and that more attention could well have been devoted to polar curves

There are a good number of exercises following the various topics and at the end of several chapters are sets of review exercises. The figures and mechanical make-up are excellent.—*P. K. Smith.*

EDITORIAL PERSONNEL OF THE MATHEMATICS NEWS LETTER

Acting upon a conviction that the steadily growing group of News Letter readers, already nation-wide in scope, are entitled to know something about the personnel of our Editorial Board, we have gathered some interesting biographic facts of which the following are offered as a third installment.—*S. T. S.*

IRBY C. NICHOLS

A native of Mississippi; B. S., University of Mississippi, 1906; M. A. ibid 1908; using History as a major subject; graduate student in mathematics, University of Chicago summer of 1911; M. S., University of Illinois, 1912, majoring in mathematics; Ph.D. University of Michigan, 1917.

Taught a number of terms in the public and high schools of Mississippi, was a Teaching Fellow and Instructor of mathematics, University of Mississippi, 1904-1908- instructor and assistant professor of mathematics in the A. & M. College of Texas, 1909-1917, except for two sessions "absent on leave" to do graduate work; Associate Professor and Professor at Louisiana State University since 1917.

Member of Phi Kappa Phi, Mathematical Association of America, charter member of Louisiana -Mississippi Section of the M. A. A., past secretary and president of this organization, and charter member of Louisiana Academy of Science, present editor of its publications.

Author "*History of Reconstruction of Desoto County, Mississippi,*" "*The Induction Into Europe of the Hindu Art of Reckoning with Fractions-*" and a number of bulletins and articles, mostly on topics of finance.

W. PAUL WEBBER

Born in Ohio where he lived until grown. Began teaching in the old fashioned country school. Attended college and taught alternately in Ohio; taught several years in Mississippi. Finally obtained Ph.D. at University of Cincinnati where he held a teaching fellowship and became an instructor. Taught in Oklahoma State College; Bethany College; University of Pittsburgh, where was acting head department several years; now at Louisiana State University. Attended University Chicago two summers; co-author Webber and Plant Introductory Mathematical Analysis; author Elementary Applied Mathematics; research in periodic functions, theory of teaching mathematics; author several articles in Mathematcis Teacher; author several bulletins on teaching published by Louisiana State University; contributor to Mathematics News Letter; member American Mathematical Society, Mathematical Association of America, Louisiana Academy of Science, listed in American Men of Science, Who's Who in American Education.